11-2 Arithmetic Series

Series is the sum of a sequence

8, 12, 16, 20, 24, sequence

8 + 12 + 16 + 20 + 24 series

$S_1 = 8$

$S_2 = 20$

$S_3 = 36$

$S_4 =$

$S_5 =$

In general, $a_1, a_2, a_3, a_4, \ldots a_n$

$S_n = a_1 + a_2 + a_3 + a_4 + \ldots + a_n$

Σ sigma summation sign

Σ

8 + 12 + 16 + 20 + 24

Find a_n .

$a_n = a_1 + (n-1)d$

$= 8 + (n-1)4$

$4n-4$

$a_n = 4n+4$

$\sum_{n=1}^{5}(4n+4)$

$$\sum_{n=1}^{5}(4n+4)$$

"the sum of 4n + 4 as n goes from 1 to 5"

Expanded form =

$8+12+16+20+24$

$$\sum_{n=3}^{7}(2n-3)$$

Put in Expanded form.

$= 3+5+7+9+11$

Put the following series into sigma notation:
3 + 6+ 9 + ... + 36

$a_n = 3 + (n-1)3$
$3n - 3$
$a_n = 3n$
$36 = 3n$
$12 = n$
$\sum_{n=1}^{12} (3n)$

Challenge:
Find the sum of the integers from 1 to 100

$1 + 100 = 101$
$2 + 99 = 101$
$3 + 98 = 101$
$\vdots$
$101 \times 50 = \frac{n}{2}(a_1 + a_n)$

Friedrich Gauss

$a_1 =$
$a_{100} =$

Sum Arith. Series $S_n = \frac{n}{2}(a_1 + a_n)$

Ex:
Find the sum of the given sequence:
5, 10, 15, 20

$S_4 = \frac{4}{2}(5 + 20)$
$= 50$

Ex:
Find the sum of the given sequence:
3, 6, 9, 12, 15, 18

$S_6 = \frac{6}{2}(3 + 18)$
$= 63$

Ex:
Find the sum of the first 50 terms of the given sequence:
3, 6, 9, 12, 15, 18

$a_n = a_1 + (n-1)d$

$S_n = \frac{n}{2}(a_1 + a_1 + (n-1)d)$

Use when you don't know last term

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$= \frac{50}{2}(2(3) + 49 \cdot 3)$

$= 3825$

When you don't have the last term, either find it or use this formula.

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

Ex:
$a_n = -14$
$n = 9$
$d = -8$
$S_9 =$ 162

$a_n = a_1 + (n-1)d$

$-14 = a_1 + (9-1)(-8)$

$50 = a_1$

$S_9 = \frac{9}{2}(50 + -14)$

HW p586-587
15-23odd, 29, 33, 35, 40